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Cancellation of finite-width divergences in threshold top-pair production at linear colliders

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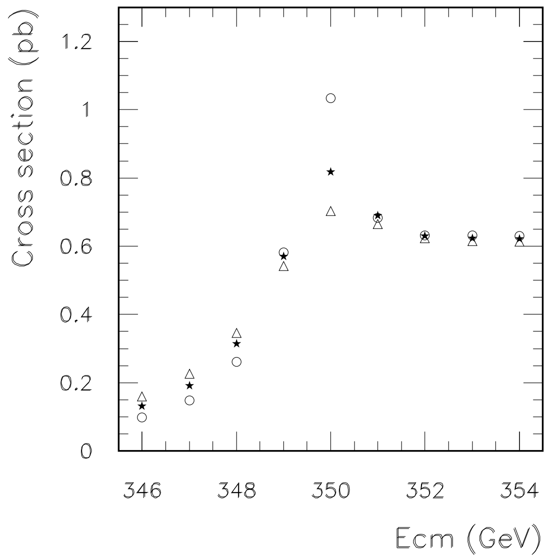
Based on Nucl. Phys. B 840 (2010) 186 [arXiv:1004.2188] *and work to be published*

I Top-pair production at linear colliders near threshold

Future linear colliders (ILC/CLIC)
with $\sqrt{s} \gtrsim 2m_t \approx 350\,\text{GeV} \rightsquigarrow$ produce many $t\bar{t}$ pairs:
clean initial state of $e^+e^- \to t\bar{t}$ allows
threshold scans with $\sqrt{s} \sim 2m_t$

$\hookrightarrow$ precise determination of top-quark parameters $(m_t, \Gamma_t, \ldots)$,
especially as input for electroweak precision observables



Martinez, Miquel '02

Need also precise theoretical prediction!

QCD corrections are known (almost) up to NNNLO/NNLL order,
but need electroweak (EW) non-resonant contributions at NLO and NNLO!

The **decay** $t\bar{t} \to (bW^+)\,(\bar{b}W^-)$ is an EW effect.
$\Rightarrow$ Describe $t\bar{t}$ production in terms of the more physical process $e^+e^- \to W^+W^-b\bar{b}$.
$\Rightarrow$ Allow for invariant-mass cuts on reconstructed $t, \bar{t}$.

Perturbative expansion: NRQCD

Decay $t \to bW^+$ with $\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\text{QCD}}$
$\hookrightarrow$ $t\bar{t}$ is perturbative at threshold. Bigi, Dokshitzer, Khoze, Kühn, Zerwas '86

Top quarks move slowly near threshold: velocity $v \sim \alpha_s \ll 1$
$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders

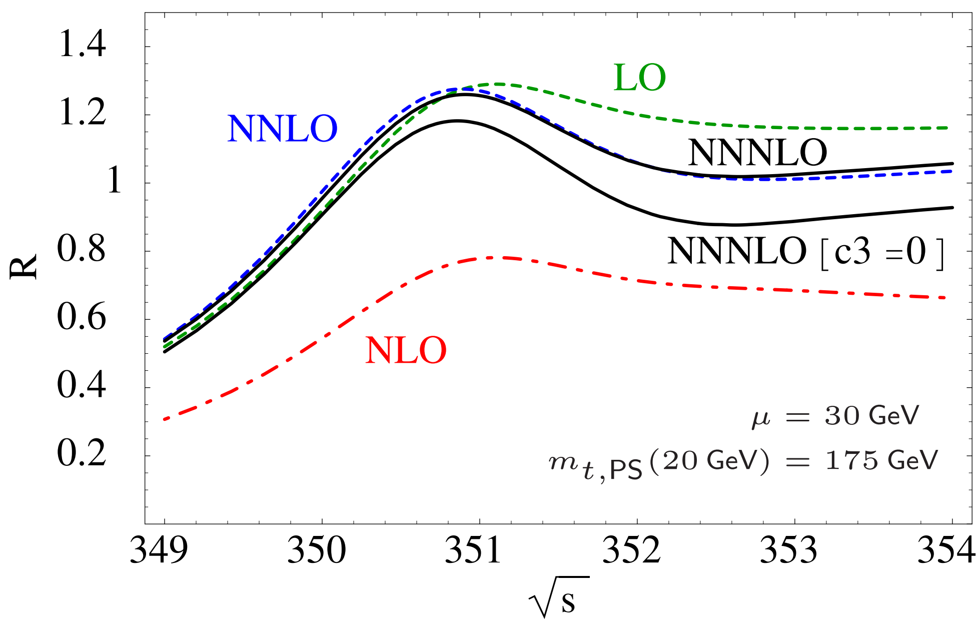
$\hookrightarrow$ expansion: LO, NLO, ... from additional powers of α_s or v:

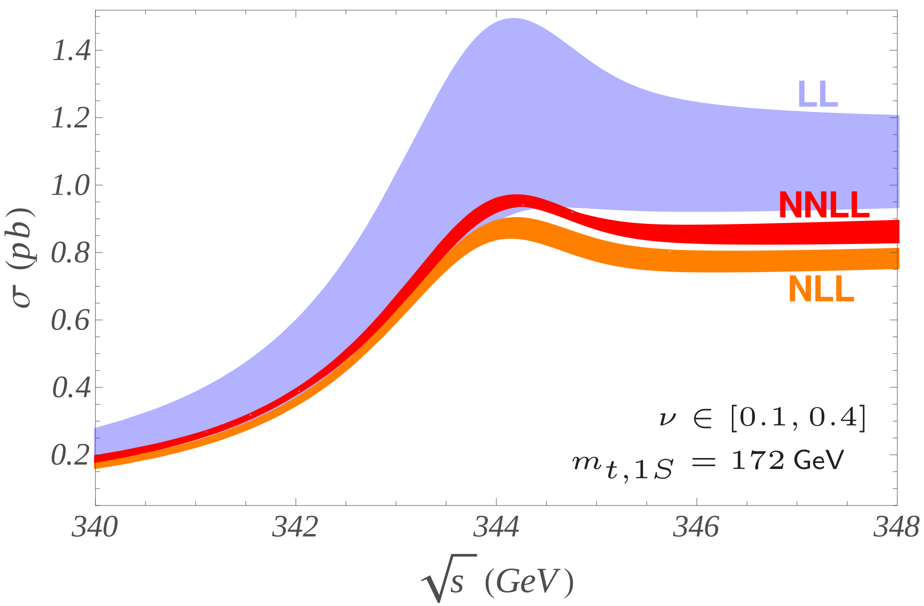
$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left(\{1\}_{\text{LO}} + \{\alpha_s, v\}_{\text{NLO}} + \{\alpha_s^2, \alpha_s v, v^2\}_{\text{NNLO}} + \dots\right)$$

Further improvement by summing also $(\alpha_s \ln v)^m$ to all orders: LL, NLL, ...

Status of QCD corrections

- NNLO QCD corrections
  Hoang, Teubner '98–'99; Melnikov, Yelkhovsky '98; Yakovlev '98; Beneke, Signer, Smirnov '99; Nagano, Ota, Sumino '99; Penin, Pivovarov '98–'99
- NNNLO (partial)
  Beneke, Kiyo, Schuller '05–'08 (⇝ left figure) [+ contributions from Kiyo, Seidel, Steinhauser '08; Anzai, Kiyo, Sumino '09; Smirnov, Smirnov, Steinhauser '09–'10]
- NNLO & NNLL
  Hoang, Manohar, Stewart, Teubner '00–'01; Hoang '03; Pineda, Signer '06;
  Stahlhofen, Hoang '11 (⇝ right figure)



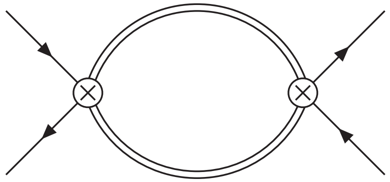


Effective field theory (EFT) for pair production of unstable particles near threshold

Beneke, Chapovsky, Khoze, Signer, Stirling, Zanderighi '01–'04;
Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07–'08

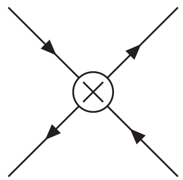
- Non-relativistic power counting: $$\alpha_s^2 \sim \alpha_{\mathrm{EW}} \sim \frac{\Gamma_t}{m_t} \sim v^2 = 1 - \frac{4m_t^2}{s}$$
- Integrate out hard modes $\sim m_t \rightsquigarrow$ EFT with potential (nearly on-shell) top quarks.
- Extract cross section $e^+e^- \to W^+W^-b\bar{b}$ from appropriate cuts of the $e^+e^- \to e^+e^-$ forward-scattering amplitude:

resonant contributions



with production operators of potential $t\bar{t}$ pair

non-resonant contributions



correspond to full-theory diagrams expanded around $\Gamma_t = 0$ and $s = 4m_t^2$

$\Rightarrow$ Potential corrections to resonant diagrams within EFT

$\Rightarrow$ Hard corrections to matching coefficients of operators

Electroweak effects at LO

- Replacement rule $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ — Fadin, Khoze '87
  (⇝ implemented in existing QCD corrections)

Electroweak effects at NLO

- Exchange of a "Coulomb photon": trivial extension of QCD corrections (available)
- Gluon exchange between top quarks and their decay products: — Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94; Hoang, Reißer '04
  ↪ cancel at NLO & NNLO in the total cross section.
  They are still negligible for *loose* top invariant-mass cuts.
- **Non-resonant corrections** ⇝ this talk

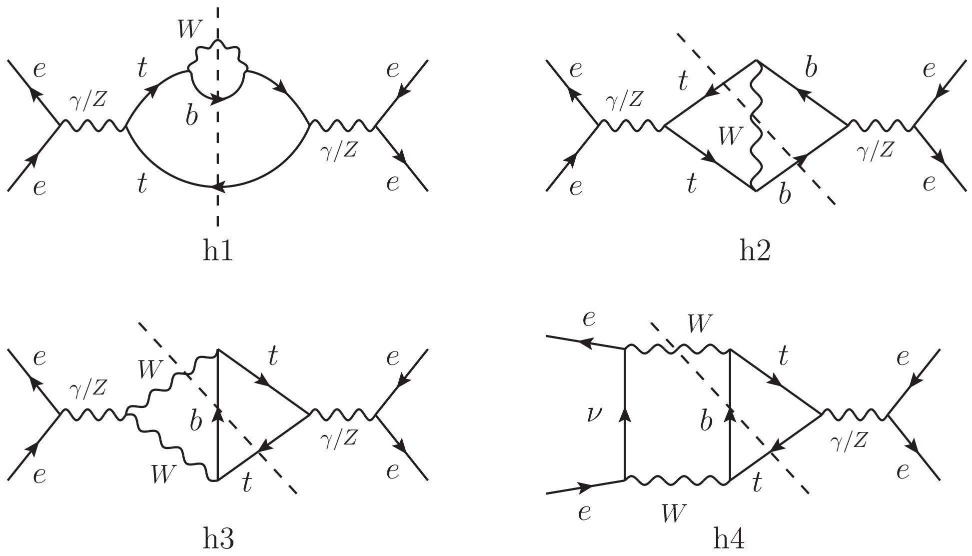
Non-resonant effects at NNLO

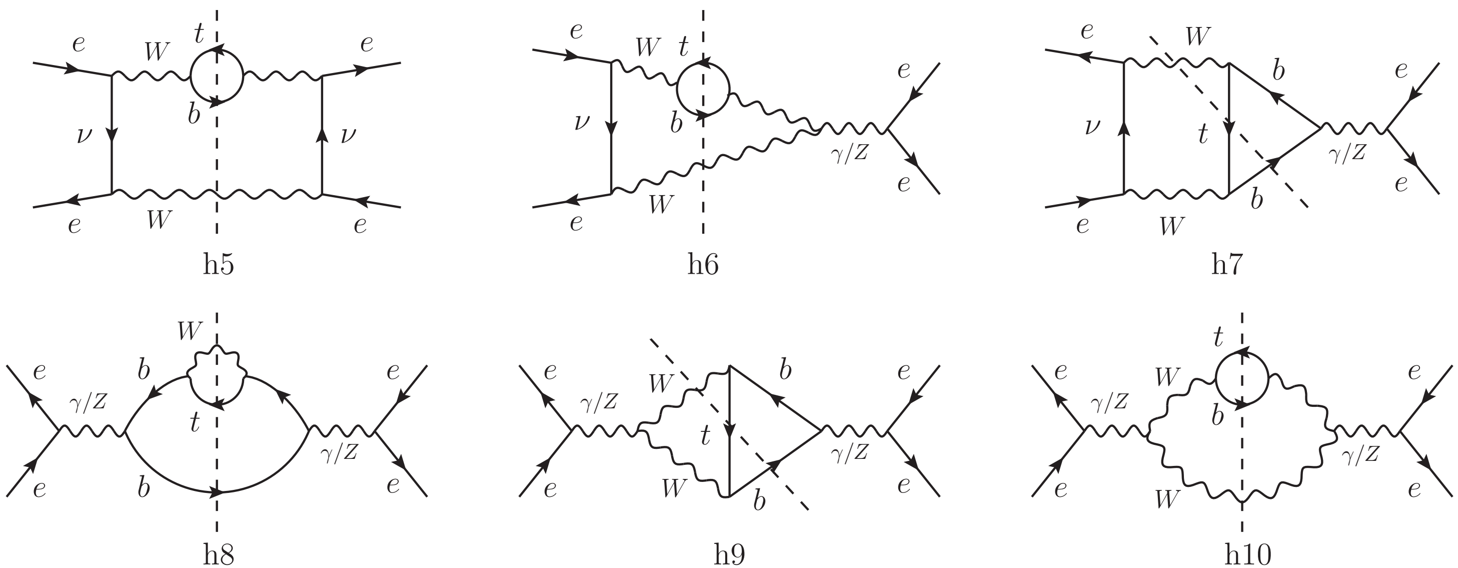
- Gluon exchange added to NLO non-resonant diagrams ⇝ this talk

II Electroweak non-resonant NLO contributions

Non-resonant corrections at NLO:

- cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward-scattering amplitude
- correspond to tree-level processes $e^+e^- \to bW^+\bar{t}$ and $e^+e^- \to \bar{b}W^-t$
- hard region at NLO: $\Gamma_t = 0$ and $s = 4m_t^2$





[symmetric diagrams not shown]

Form of non-resonant contributions

With the reconstructed top momentum $p_t = p_b + p_{W^+}$ (top only present in h_1–h_4), the contributions of all diagrams (for $s = 4m_t^2$) are of the form:

$$\int_{\Delta^2}^{m_t^2} \mathrm{d}p_t^2 \, (m_t^2 - p_t^2)^{1/2-\epsilon} \, H_i\bigg(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\bigg)$$

Total cross section: $\Delta^2 = M_W^2$

Top invariant-mass cuts:

Restrict invariant masses $M_{t,\bar{t}}$ of the reconstructed $t, \bar{t}$: $\boxed{\big|M_{t,\bar{t}} - m_t\big| \leq \Delta M_t}$

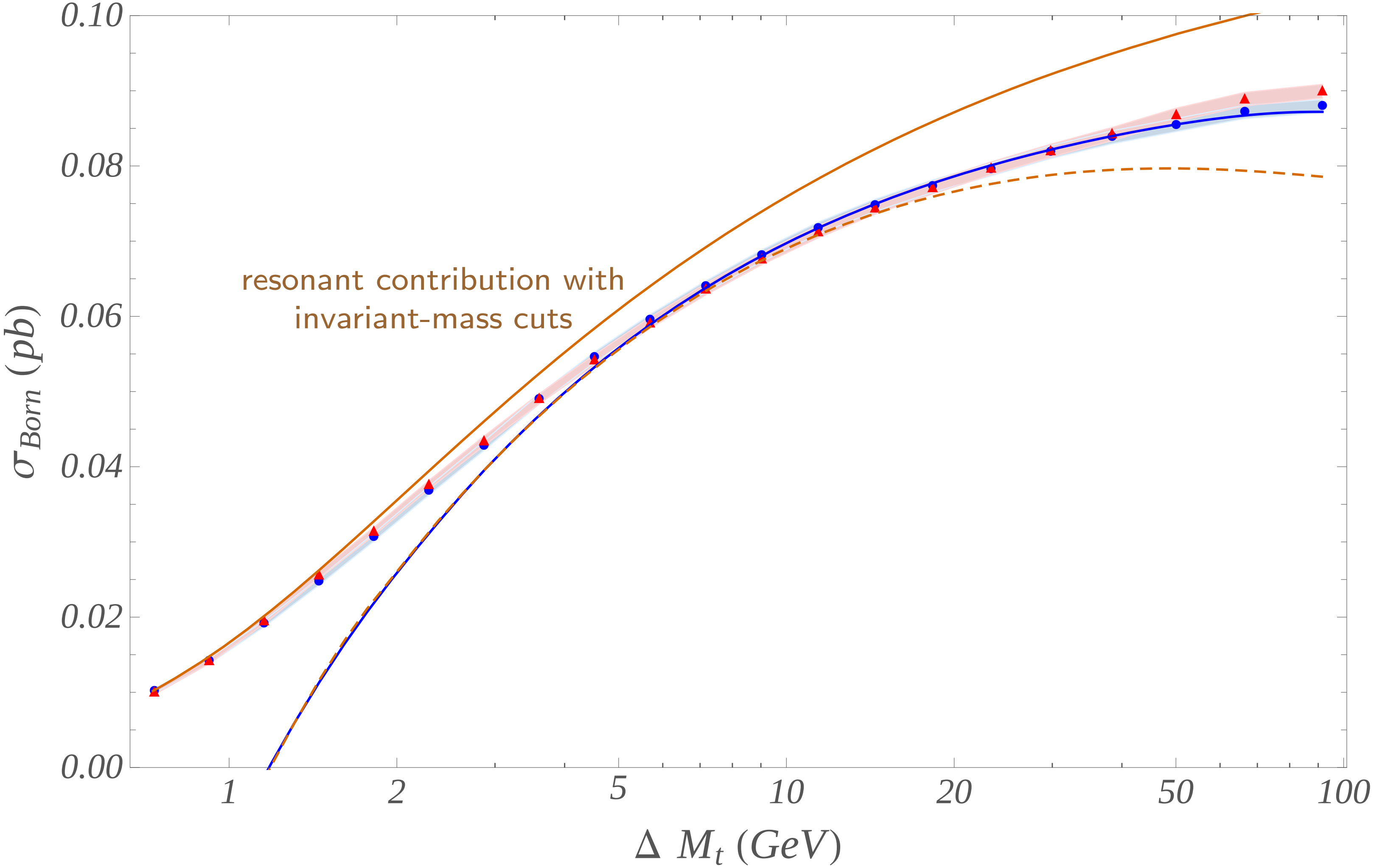
$\hookrightarrow$ lower integration limit $\Delta^2 = m_t^2 - \Lambda^2$ with $\boxed{\Lambda^2 = (2m_t - \Delta M_t)\, \Delta M_t}$

We focus on loose cuts with $\Delta M_t \gg \Gamma_t \iff \Lambda^2 \gg m_t \Gamma_t$

$\hookrightarrow$ no cut needed for resonant contributions.

EW tree-level contributions: cut-dependence at threshold

cross section (for $\alpha_s = 0$) at threshold ($s = 4m_t^2$) as a function of the invariant-mass cut ΔM_t



full MG points,
MG without Higgs,
our result,
HRR result

total cross section reached for $\Delta M_t = 91.6$ GeV

Our result (solid-blue): EW non-resonant NLO + resonant NNLO tree-level contributions

↪ good agreement with MadGraph (MG) for loose cuts $\Delta M_t \gtrsim 5$ GeV ✓

HRR result [Hoang, Reißer, Ruiz-Femenía '10]: dashed-brown ⇒ agreement for small ΔM_t ✓

Our results confirmed (even diagram-wise) by approach of [Penin, Piclum '11] ✓

III NNLO contributions

Finite-width divergences in resonant contributions

Resonant contributions expanded for potential (nearly on-shell) top quarks,
but integrated over all momenta $\rightsquigarrow$ uncancelled UV singularity from hard momenta!
$\hookrightarrow$ Cancellation with non-resonant (hard) contributions @ potential momenta.

Beneke, Kiyo '08

UV divergences are related to finite top width Γ_t via cut through NRQCD propagator:

- for stable top $\to \pi\,\delta\big(p^0 - \frac{\vec{p}^{\,2}}{2m_t}\big)$,
- for unstable top $\to \frac{\Gamma_t/2}{(p^0 - \vec{p}^{\,2}/2m_t)^2 + (\Gamma_t/2)^2}$ Breit–Wigner, UV-behaviour changed!

At **NNLO**: finite-width divergences $\propto \boxed{\alpha_s \frac{\Gamma_t}{\epsilon}}$ (in dimensional regularization)

$$\begin{aligned} \text{div}\, \sigma_{\text{res}}^{\text{NNLO}} = & \Big[(C_p^{(v)})^2 + (C_p^{(a)})^2\Big]\, 2N_{\text{c}}\, \text{div}\, \big[\text{Im}\, G_{\dots}^{(2)}\big] \\ & + \Big[(C_{p,P\text{-wave}}^{(v)})^2 + (C_{p,P\text{-wave}}^{(a)})^2\Big] \frac{4N_{\text{c}}}{3m_t^2}\, \text{div}\, \big[\text{Im}\, G_{P\text{-wave}}^{(2)}\big] \\ & + \Big[C_p^{(v)}\, C_p^{(v),\text{abs}} + C_p^{(a)}\, C_p^{(a),\text{abs}}\Big]\, 4N_{\text{c}}\, \text{div}\, \big[\text{Re}\, G_{\text{C}}^{(0)}\big] \end{aligned}$$

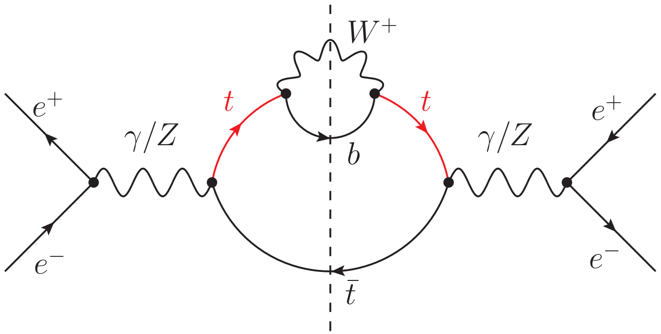
Hoang, Reißer '04

Endpoint divergences in non-resonant contributions

$\hookrightarrow$ cancel finite-width divergences

Endpoint divergences of the phase-space integration at $p_t^2 \to m_t^2$ (because $\Gamma_t = 0$ here):

NLO:

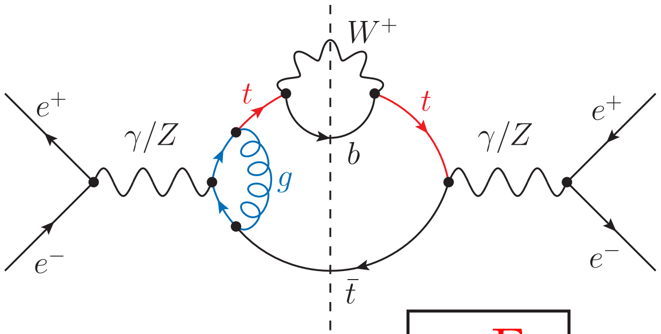


$$\sim \int^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{n+\epsilon}} \text{ with } n = \tfrac{3}{2}, \tfrac{1}{2}, \ldots$$

$\hookrightarrow$ endpoint divergence finite in dim. reg.:

$$\int_{m_t^2-\Lambda^2}^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{\frac{3}{2}+\epsilon}} = -\frac{2}{\Lambda} + \mathcal{O}(\epsilon)$$

NNLO:



$$\sim \int^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{n+a\epsilon}} \text{ with } n = 2, \tfrac{3}{2}, \mathbf{1}, \tfrac{1}{2}, \ldots$$

$\hookrightarrow$ endpoint divergence $\propto \boxed{\alpha_s \frac{\Gamma_t}{\epsilon}}$ from $n = 1$:

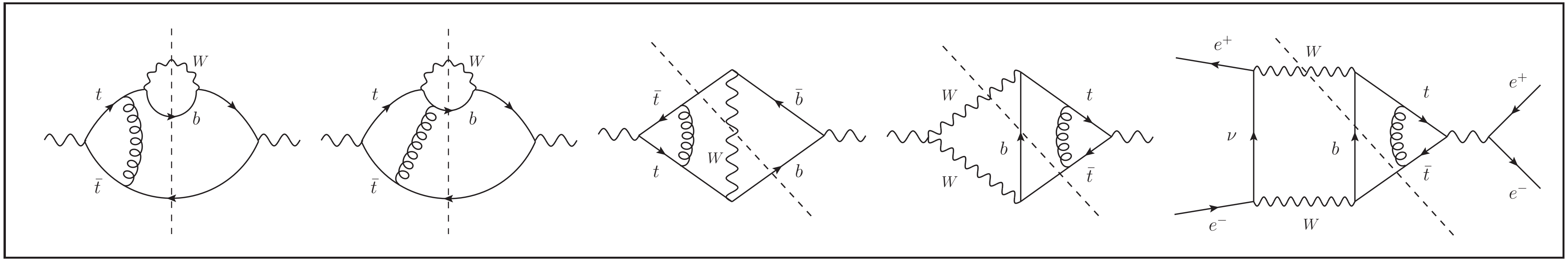
$$\mu^{4\epsilon} \int_{m_t^2-\Lambda^2}^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{1+2\epsilon}} = -\frac{1}{2\epsilon} + \ln\frac{\Lambda^2}{\mu^2} + \mathcal{O}(\epsilon)$$

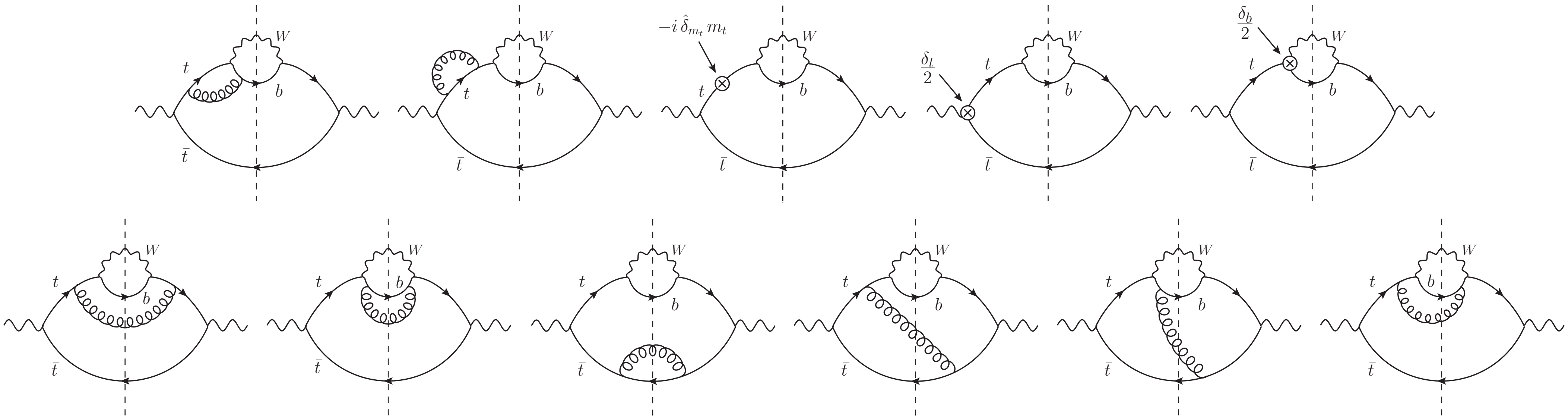
Expand integrand in $(m_t^2 - p_t^2)/m_t^2 \iff$ asymptotic expansion of result in Λ/m_t

Endpoint-divergent non-resonant NNLO diagrams

[symmetric diagrams not shown]

$\hookrightarrow$ expanded near endpoint $\rightsquigarrow$ potential top momentum $p_t = p_b + p_W (+p_g)$





boxed diagrams $\rightsquigarrow$ endpoint-singular $\frac{1}{\epsilon} - 2\ln\frac{\Lambda^2}{\mu^2}$ terms from potential gluons

$+$ "finite" endpoint-divergent $\frac{m_t}{\Lambda}$ & $\frac{m_t^2}{\Lambda^2}$ terms from hard & potential gluons

(total ultrasoft contribution cancelled like in NNLO resonant contributions)

Endpoint-divergent non-resonant NNLO result

$\hookrightarrow$ dominant contribution for small Λ (or small ΔM_t)

$$\left[\begin{array}{c} x = M_W^2/m_t^2 \\ C_{\dots}(s) = \gamma/Z\text{-prop. \& } e^\pm\text{-coupl.}\end{array}\right]$$

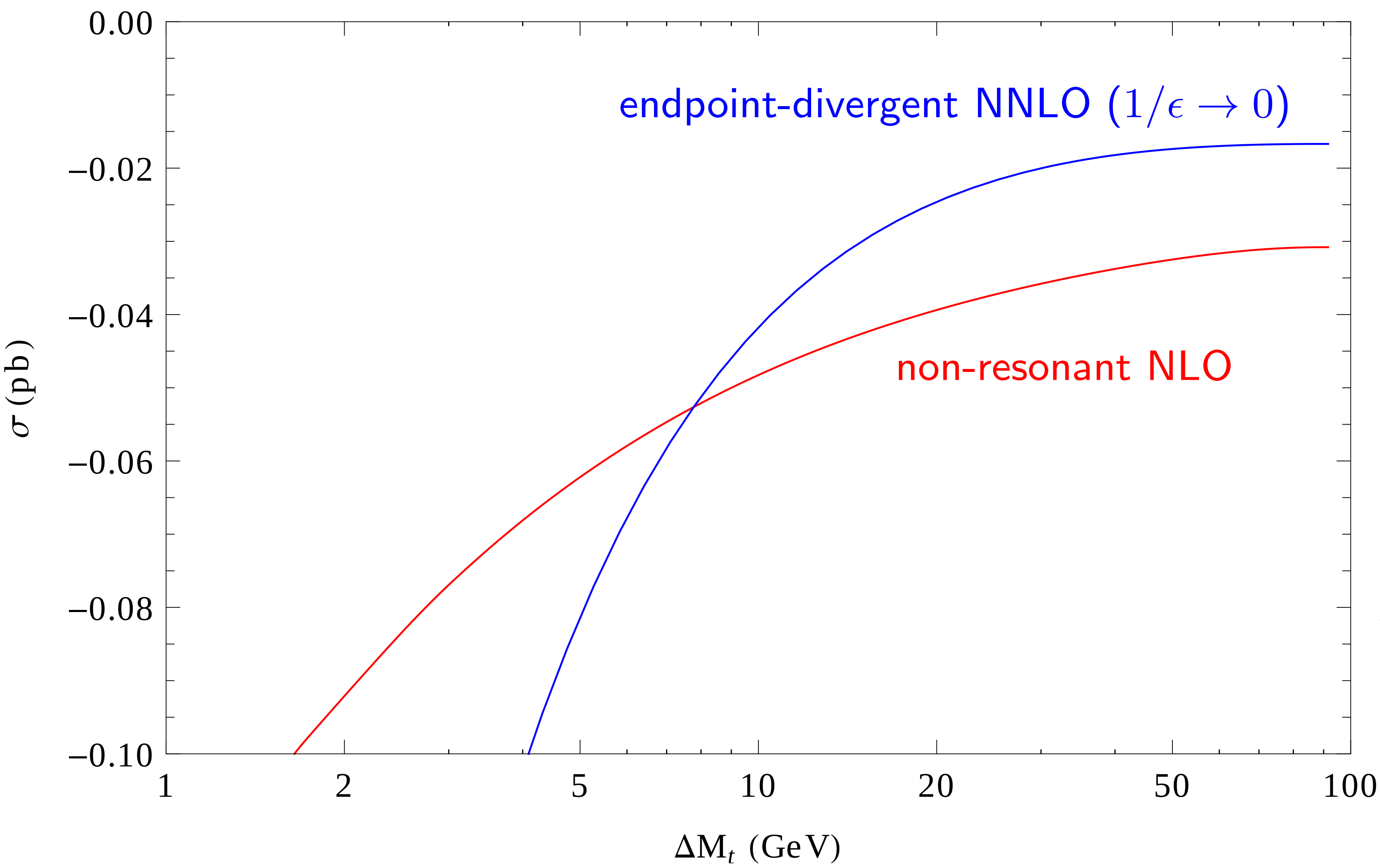
$$\begin{aligned}
\sigma^{\text{NNLO}}_{\text{non-res}} = \frac{64\pi^2\alpha^2}{s}\,\frac{\Gamma_t}{m_t}\,N_c C_F\,\frac{\alpha_s}{4\pi}\Bigg\{&\Big[Q_t^2\,C_{\gamma\gamma}(s) - 2Q_t v_t\,C_{\gamma Z}(s) + v_t^2\,C_{ZZ}(s)\Big]\Bigg\{4\,\frac{m_t^2}{\Lambda^2} \\
&- \frac{m_t}{\Lambda}\,\frac{\sqrt{2}}{\pi^2}\left[2\left(2\ln x + \frac{5+4x}{1+2x}\right)\ln(1-x) + 8\,\text{Li}_2(x) + \frac{4\pi^2}{3}\right. \\
&\left.+ \frac{4x\,(1+x)\,(1-2x)}{(1-x)^2\,(1+2x)}\,\ln x + \frac{11+7x-26x^2}{(1-x)\,(1+2x)}\right]\Bigg\} \\
+ \left(\frac{1}{\epsilon} - 2\ln\frac{\Lambda^2}{\mu^2}\right)\Bigg\{&-\frac{7+7x+22x^2}{6\,(1-x)\,(1+2x)}\Big[Q_t^2\,C_{\gamma\gamma}(s) - 2Q_t v_t\,C_{\gamma Z}(s) + v_t^2\,C_{ZZ}(s)\Big] \\
&+ \frac{1}{3}\,a_t^2\,C_{ZZ}(s) + \frac{1}{2}\Big[Q_t a_t\,C_{\gamma Z}(s) - v_t a_t\,C_{ZZ}(s)\Big] + \frac{1-5x-2x^2}{6\,(1+x)\,(1+2x)}\times \\
&\times\Big[Q_t Q_b\,C_{\gamma\gamma}(s) - \big(Q_t\,(v_b+a_b) + Q_b v_t\big)\,C_{\gamma Z}(s) + v_t\,(v_b+a_b)\,C_{ZZ}(s)\Big] \\
&+ \frac{2+5x-2x^2}{6x\,(1+2x)}\left[Q_t\,C_{\gamma\gamma}(s) - \left(v_t + Q_t\,\frac{c_W}{s_W}\right)C_{\gamma Z}(s) + v_t\,\frac{c_W}{s_W}\,C_{ZZ}(s)\right] \\
&- \frac{Q_t\,C_\gamma(s) + v_t\,C_Z(s)}{4\,(1-x)^3\,(1+2x)}\left[x\,\ln\left(\frac{2}{x}-1\right) + \frac{(1-x)\,(1-2x-23x^2)}{12x}\right]\Bigg\}\Bigg\} \\
+ \mathcal{O}(\epsilon) + \text{finite }&\Lambda\text{-independent terms} + \mathcal{O}(\Lambda/m_t)
\end{aligned}$$

- UV and IR singularities cancelled between diagrams ✓
- $1/\epsilon$ endpoint singularities & finite-width divergences cancel each other ✓
- comparison to HRR result: m_t^2/Λ^2 ✓, $\Lambda^0\,\ln(\Lambda^2)$ ✓, m_t/Λ absent there

Hoang, Reißer, Ruiz-Femenía '10

Non-resonant NLO & NNLO contributions: cut-dependence at threshold

[$\mu = 30$ GeV]

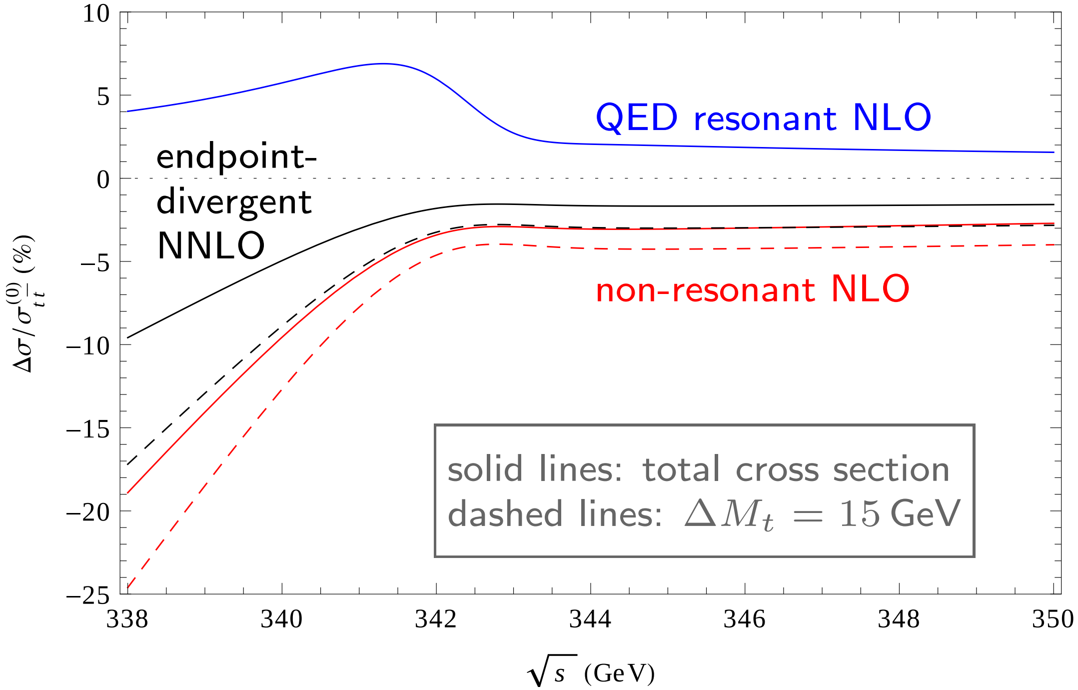


total cross section reached for $\Delta M_t = 91.6$ GeV

Perturbative expansion converges for loose cuts:

$$\alpha_s \frac{m_t^2}{\Lambda^2} \text{ @ NNLO} \ll \frac{m_t}{\Lambda} \text{ @ NLO} \quad \Longleftrightarrow \quad \Lambda^2 \gg m_t\Gamma_t \sim m_t^2\,\alpha_{\text{EW}} \sim m_t^2\,\alpha_s^2$$

Full cross section with QCD LO + QED NLO + non-resonant contributions



$[m_t = 172\,\text{GeV},\ \mu = 30\,\text{GeV}]$

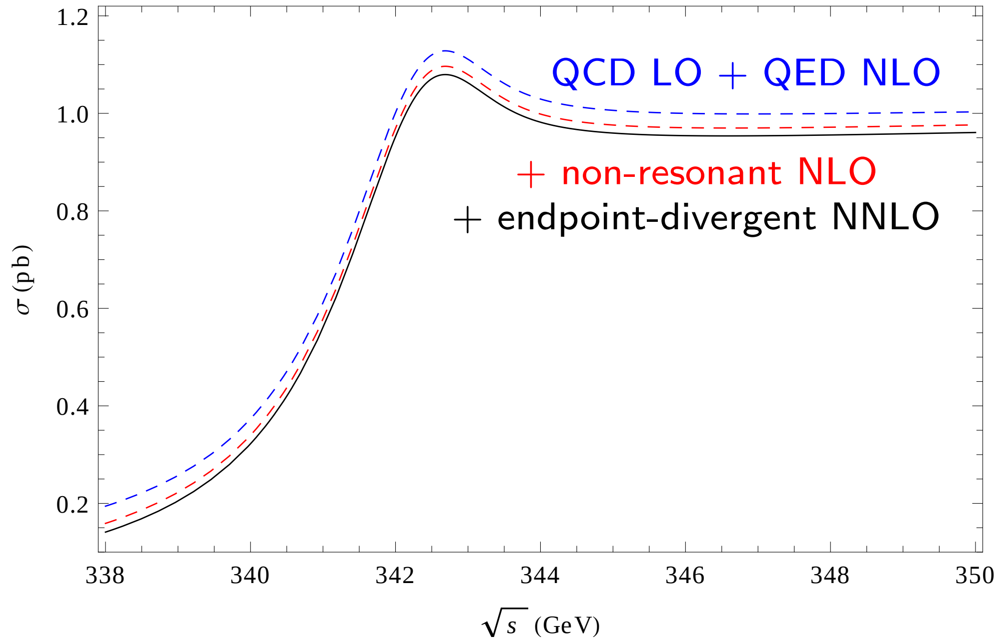
Relative sizes of corrections w.r.t. LO QCD (incl. resummed "Coulomb gluons"):

QED resonant correction ("Coulomb photons"),

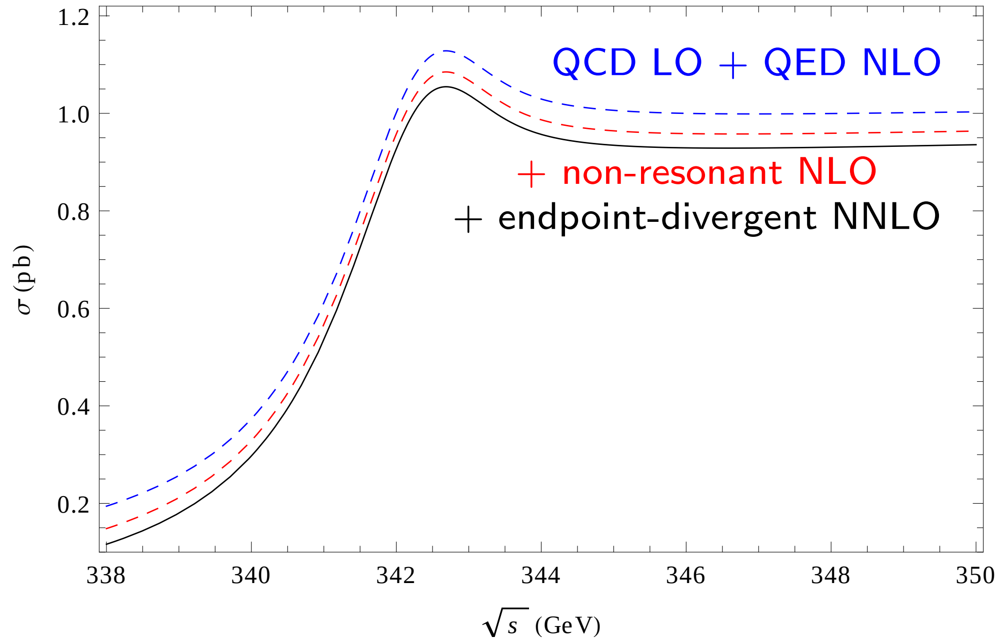
non-resonant NLO correction,

endpoint-divergent non-resonant NNLO correction (dominant terms for small ΔM_t)

Total cross section



Cross section with $\Delta M_t = 15$ GeV



IV Summary & outlook

Non-resonant contributions to threshold top-pair production at linear colliders

- NLO evaluated for total cross section and with top invariant-mass cuts ΔM_t:
  $\Delta\sigma^{\text{NLO}}_{\text{tot}} =$ -31 fb (-3.1%), $\Delta\sigma^{\text{NLO}}_{\Delta M_t = 15\,\text{GeV}} =$ -42 fb (-4.2%) at threshold
- endpoint-divergent NNLO contributions $\rightsquigarrow$ dominant terms for small ΔM_t:
  $\Delta\sigma^{\text{NNLO}}_{\text{tot}} =$ -17 fb (-1.7%), $\Delta\sigma^{\text{NNLO}}_{\Delta M_t = 15\,\text{GeV}} =$ -30 fb (-3.0%) at threshold

$\hookrightarrow$ improve accuracy of NRQCD prediction

Singularities of NNLO contributions

- finite-width divergences from resonant contributions $\propto \alpha_s \Gamma_t/\epsilon$
- endpoint divergences from non-resonant contributions $\propto \alpha_s \Gamma_t/\epsilon$

$\hookrightarrow$ mutual cancellation shown ✓

Outlook

- complete non-resonant NNLO contributions (numerically)
- consistent addition with resonant NNLO contributions

Extra slides

NLO results & comparisons

obtained with $m_t = 172\,\text{GeV}$ and $\Gamma_t = \Gamma_t^{\text{tree}} = 1.46550\,\text{GeV}$

Tree-level comparison to MadGraph (MG)

Alwall et al. '07

- generated 10^4 events for $e^+e^- \to W^+W^-b\bar{b}$,
- analyzed dependence on invariant-mass cuts

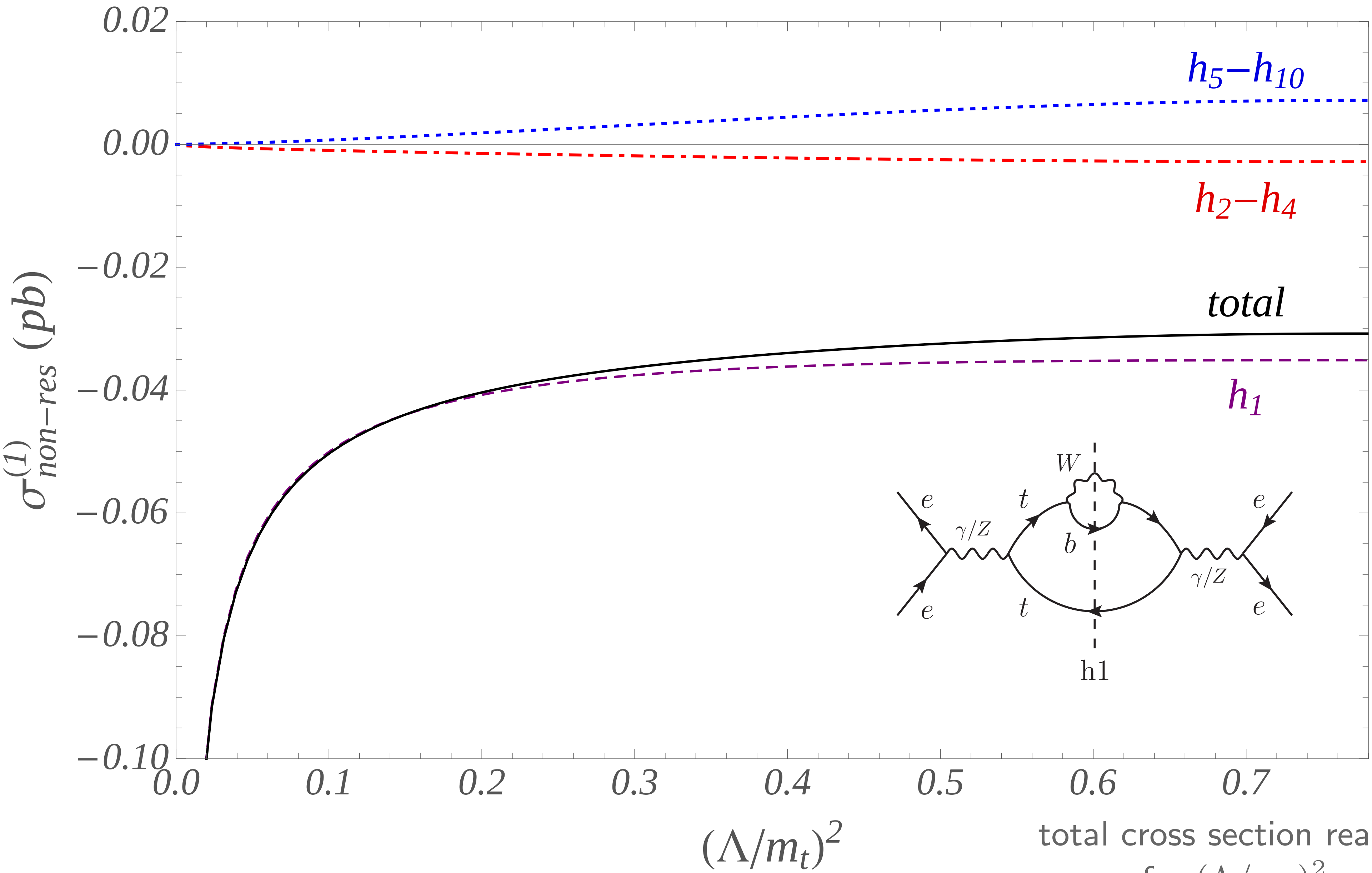
Comparison to alternative approach

Hoang, Reißer, Ruiz-Femenía '10

- invariant-mass cuts through "phase-space matching" within non-relativistic EFT (QCD & EW @ NLO + some NNLO contributions)
- contributions are expanded for moderate invariant-mass cuts
  $15\,\text{GeV} \leq \Delta M_t \leq 35\,\text{GeV}$
  $\leftrightarrow$ our result is also valid for larger ΔM_t up to the total cross section.
- EW contributions match leading powers in Λ/m_t of our result
  $\hookrightarrow$ agreement for small cut parameter ΔM_t or Λ

NLO non-resonant corrections: contributions of the diagrams

contribution to cross section as a function of the invariant-mass cut $m_t^2 - p_t^2 \leq \Lambda^2$

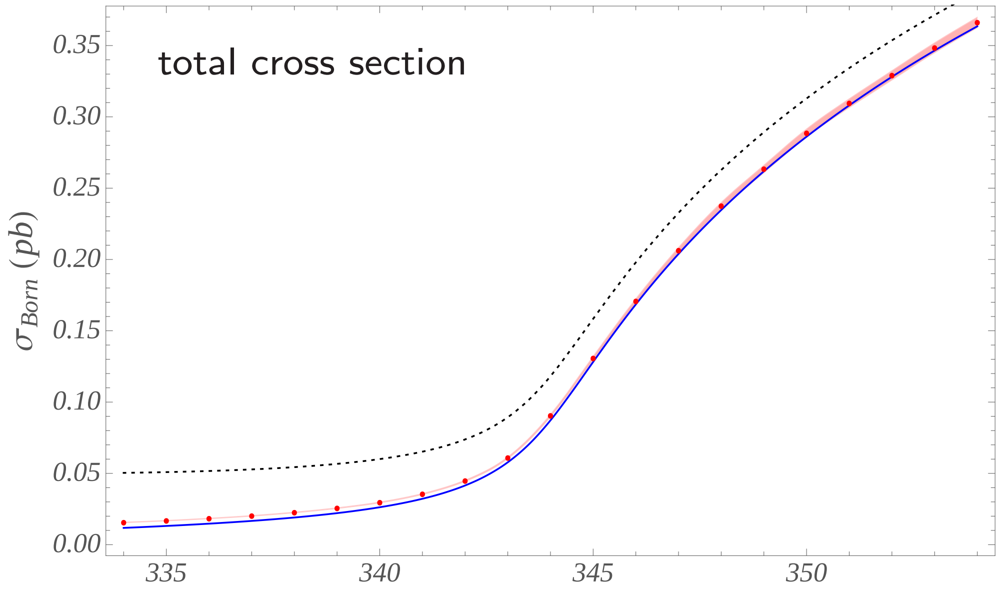


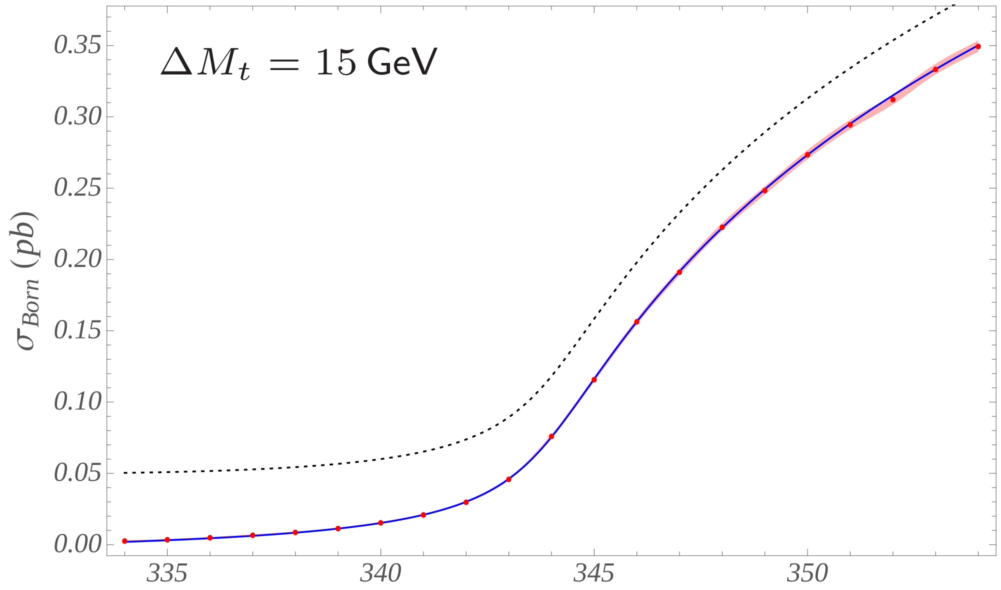
total cross section reached for $(\Lambda/m_t)^2 = 0.78$

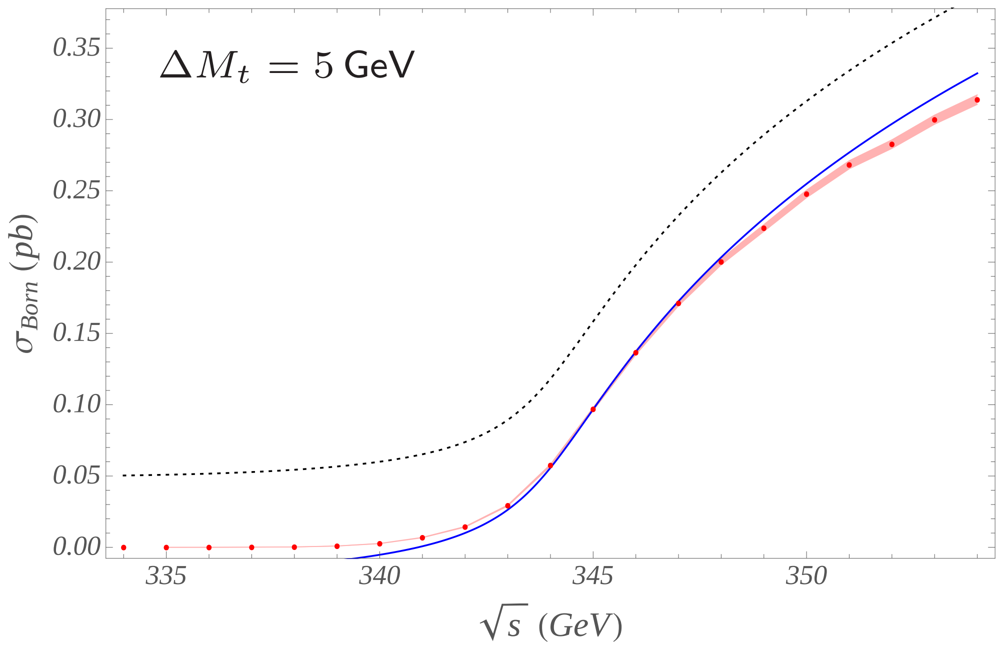
$\hookrightarrow$ diagram h_1 yields dominant contribution

EW tree-level contributions: energy-dependence for different cuts

cross section (for $\alpha_s = 0$) as a function of the centre-of-mass energy $\sqrt{s}$







MG (full) points & error band,

EW NNLO tree-level contributions (solid-blue) [resonant + non-resonant],

only resonant contributions (dotted-black)