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Project **A1**

Electroweak Sudakov logarithms

New results for the form factor in a massive U(1) model and U(1)$\times$U(1) with mass gap

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I Why logarithmic 2-loop results in EW theory?
II Massive U(1) form factor: evolution equation & 2-loop results
III U(1)$\times$U(1) model with mass gap: factorization of IR singularities
IV Summary & outlook

I Why logarithmic 2-loop results in EW theory?

Electroweak (EW) precision physics

- experimentally measured by now at energy scales up to $\sim M_{W,Z}$
- future generation of accelerators (LHC, LC) $\rightarrow$ TeV region

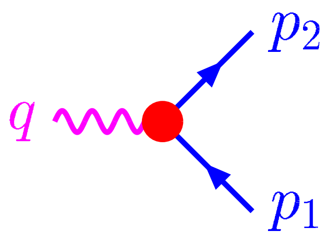
Electroweak radiative corrections
at high energies $\sqrt{s} \sim$ TeV $\gg M_{W,Z}$

Kühn et al. '00, '01; Fadin et al. '00; Denner et al. '01, '03; B.F. et al. '03; Pozzorini '04

large negative corrections in *exclusive* cross sections

EW corrections dominated by Sudakov logarithms $\alpha^n \ln^{2n}(s/M_W^2)$

Form factor F of vector current:



$$= \bar{u}(p_2)\,\gamma^\mu\,u(p_1)\cdot F + \ldots$$

High energy behaviour → *Sudakov limit*

- momentum transfer $|q^2| \equiv Q^2 \gg M^2$
- neglect fermion masses
- *logarithmic approximation*: neglect terms suppressed by a factor of M^2/Q^2

  ↪ works well for 2-loop n_f contribution where the exact result in M^2/Q^2 is known

  B.F., Kühn, Moch '03

Simplified models

Standard Model

SU(2)$_L$ ⊗ U(1)

$M_W \neq M_Z$, $M_\gamma = 0$

⟶

U(1)$_M$ ⊗ U(1)$_\lambda$ model

2 *Abelian* interactions

mass gap $M \gg \lambda \to 0$

⟶

massive U(1) model

Abelian (∼ QED)

gauge boson mass M

II Massive U(1) form factor

Form factor in perturbation theory: $F = 1 + \alpha\, F_1 + \alpha^2\, F_2 + \ldots$

large radiative corrections for $Q \sim$ TeV $\rightarrow$ sum up large logarithms to all orders in α

Evolution equation in logarithmic approximation: Sen '81; Collins '89; Korchemsky '89; ...

$$\frac{\partial F(Q^2)}{\partial \ln Q^2} = \left[\int_{M^2}^{Q^2} \frac{dx}{x} \gamma\big(\alpha(x)\big) + \zeta\big(\alpha(Q^2)\big) + \xi\big(\alpha(M^2)\big) \right] F(Q^2)$$

Solution $\rightarrow$ resummation (schematically):

$$F = 1 + \alpha\,(\ln^2 + \ln + \text{const}) + \alpha^2\,(\ln^4 + \ln^3 + \ln^2 + \ln + \text{const}) + \ldots$$

$$\rightarrow (1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots)\, \exp\Big(\alpha\,(\ln^2 + \ln) + \alpha^2\,(\ln^3 + \ln^2 + \ln) + \ldots\Big)$$

Anomalous dimensions γ, ζ, ξ from 1-loop calculation & massless 2-loop result

$\Rightarrow$ NNLL approximation of F_2 known: $\alpha^2\,(\ln^4 + \ln^3 + \ln^2)$

Massive U(1) form factor in 2-loop approximation

Known from resummation & full calculation of n_f contribution: ($n_f = \#$ fermions)

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \left[+\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right) - \left(\frac{4}{9}n_f + 3\right)\ln^3\left(\frac{Q^2}{M^2}\right) + \left(\frac{38}{9}n_f + \frac{2}{3}\pi^2 + 8\right)\ln^2\left(\frac{Q^2}{M^2}\right) - \left(\frac{34}{3}n_f + \ldots\right)\ln\left(\frac{Q^2}{M^2}\right) + \left(\frac{16}{27}\pi^2 + \frac{115}{9}\right)n_f + \ldots\right]$$

Kühn, Moch, Penin, Smirnov '01
B.F., Kühn, Moch '03

- growing coefficients with alternating sign:

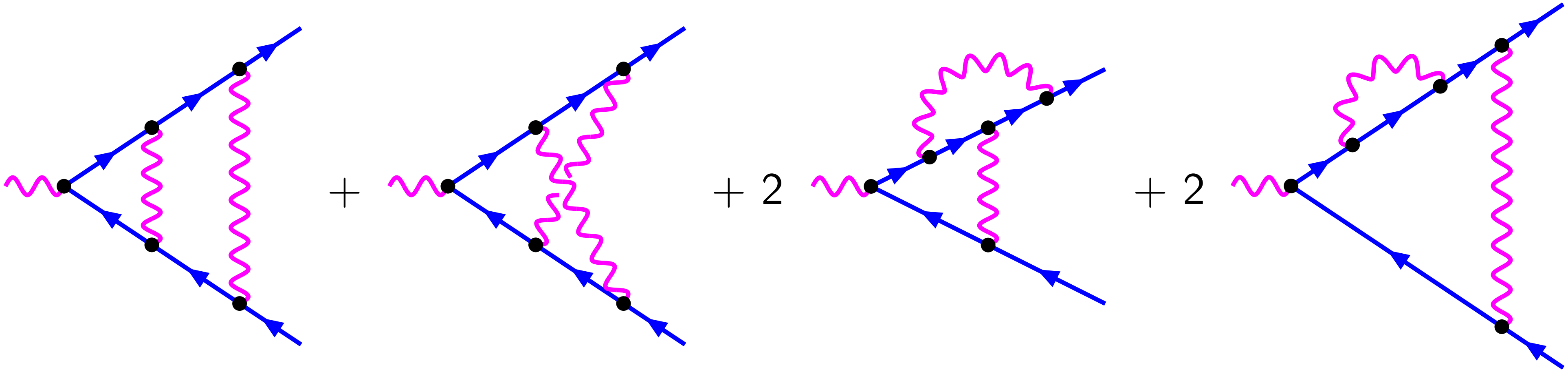
$$\begin{array}{rrrrr} & -\,0.4\,n_f\ln^3 & +\,4.2\,n_f\ln^2 & -\,11.3\,n_f\ln & +\,18.6\,n_f \\ +\,0.5\ln^4 & -\;3\;\ln^3 & +\;14.6\;\ln^2 & -\;\ldots\;\ln & +\;\ldots \end{array}$$

- $Q \sim 1\,\text{TeV} \rightarrow +\ln^4 \sim -\ln^3 \sim +\ln^2$
  $\rightarrow$ large cancellations between logarithmic terms

Complete 2-loop corrections in logarithmic approximation necessary.

Massive U(1) form factor in 2-loop approximation: calculation $(n_f = 0)$

- complete 2-loop result $\rightarrow$ loop calculation (*independent* of evolution equation)
- 2-loop vertex diagrams (massless fermions, massive bosons, 1 external scale):



+ external leg corrections + 1-loop $\times$ 1-loop

- evaluation of scalar diagrams: expansion by regions — Beneke, Smirnov '97
- algorithms in FORM & Mathematica

Massive U(1) form factor in 2-loop approximation: result $(n_f = 0)$

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \Bigg[$$

$$+\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right)$$

agreement ✓

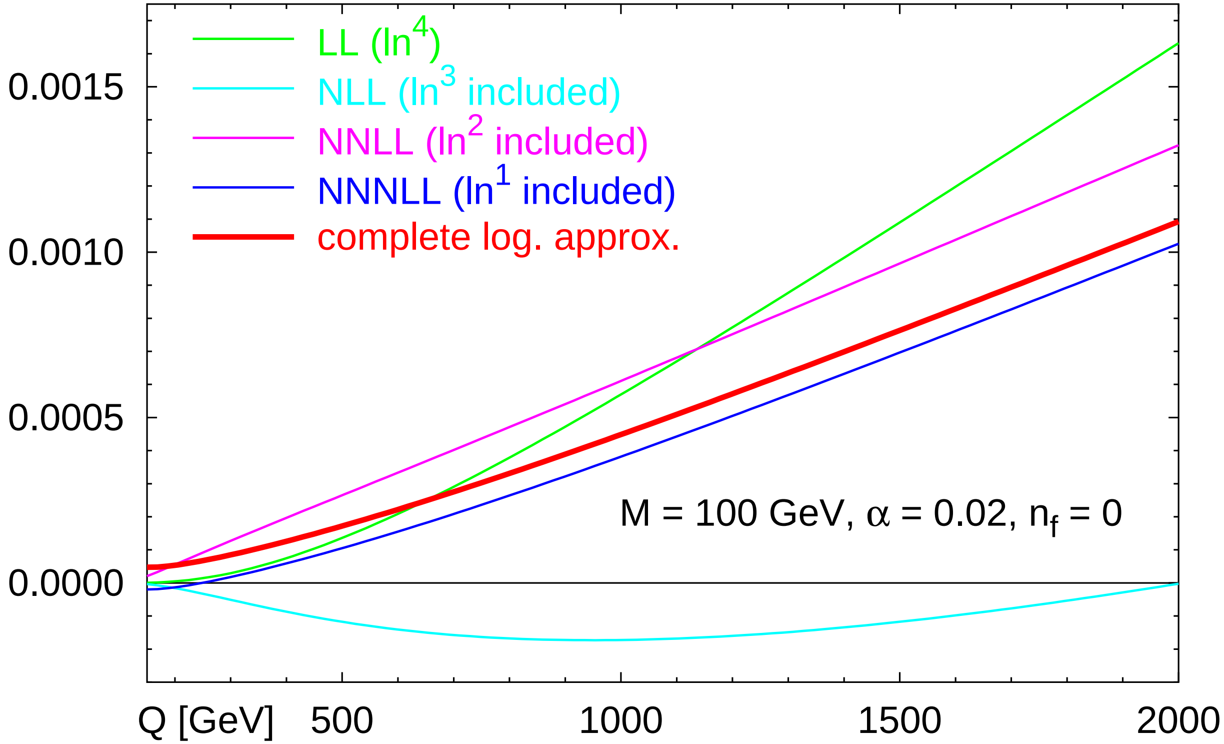
$$-3\ln^3\left(\frac{Q^2}{M^2}\right)$$

$$+\left(\frac{2}{3}\pi^2+8\right)\ln^2\left(\frac{Q^2}{M^2}\right)$$

$$-\left(-24\zeta_3+4\pi^2+9\right)\ln\left(\frac{Q^2}{M^2}\right)$$

$$+256\,\mathrm{Li}_4\left(\frac{1}{2}\right)+\frac{32}{3}\ln^4 2-\frac{32}{3}\pi^2\ln^2 2-\frac{52}{15}\pi^4+80\zeta_3+\frac{52}{3}\pi^2+\frac{25}{2}\Bigg]$$

new!



size of coefficients:	$+0.5\ln^4 - 3\ln^3 + 14.6\ln^2 - 19.6\ln + 26.4$
at $Q = 1$ TeV:	$+225 \quad -293 \quad +309 \quad -90.4 \quad +26.4$

III U(1)×U(1) model with mass gap

EW theory: massive and massless gauge bosons

↪ consider $\mathrm{U}(1)_M \times \mathrm{U}(1)_\lambda$ model with 2 different masses $M \gg \lambda \to 0$

- pure $\mathrm{U}(1)_M$: form factor $F(\alpha, Q, M)$
- pure $\mathrm{U}(1)_\lambda$: form factor $F(\alpha', Q, \lambda)$
  → known from massive U(1) result ($M \to \lambda$, $\alpha \to \alpha'$)
  → IR singularities regularized by λ
- combined $\mathrm{U}(1)_M \times \mathrm{U}(1)_\lambda$: $\hat{F}(\alpha, \alpha', Q, M, \lambda)$ for $Q \gg M \gg \lambda \to 0$
  → **Factorization of IR singularities:**

$$\hat{F}(\alpha, \alpha', Q, M, \lambda) = \underbrace{F(\alpha', Q, \lambda)}_{\text{IR singular}} \underbrace{\tilde{F}(\alpha, \alpha', Q, M)}_{\text{IR finite}} + \mathcal{O}\left(\alpha\alpha' \frac{\lambda^2}{M^2}\right)$$

Factorization of U(1)×U(1) form factor: results $(n_f = 0)$

$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M) + \mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$$

$$\Rightarrow \tilde{F}(\alpha,\alpha',Q,M) = \lim_{\lambda\to 0}\frac{\hat{F}(\alpha,\alpha',Q,M,\lambda)}{F(\alpha',Q,\lambda)} = \lim_{\varepsilon\to 0}\frac{\hat{F}_\varepsilon(\alpha,\alpha',Q,M,0)}{F_\varepsilon(\alpha',Q,0)}$$

$\hookrightarrow$ set $\lambda = 0$ and calculate $\hat{F}_\varepsilon(\alpha,\alpha',Q,M,0)$ in dimensional regularization

Calculation of 2-loop diagrams with 1 massive and 1 massless gauge boson $\longrightarrow$

$$\tilde{F}(\alpha,\alpha',Q,M) = F(\alpha,Q,M)\times$$

$$\left\{1 + \frac{\alpha\alpha'}{(4\pi)^2}\left[\underbrace{\left(48\zeta_3 - 4\pi^2 + 3\right)}_{21.2}\ln\left(\frac{Q^2}{M^2}\right) + \underbrace{\frac{7}{45}\pi^4 - 84\zeta_3 + \frac{20}{3}\pi^2 - 2}_{-22.0}\right]\right\}$$

$\Rightarrow$ interference terms are finite $\rightsquigarrow$ IR singularities factorize

$\Rightarrow$ additional terms contain only single logarithm $\ln^1$

IV Summary & outlook

Massive U(1) form factor

- simple model with massive gauge bosons
- complete 2-loop result in logarithmic approximation ✓

U(1)×U(1) model with mass gap

- first step towards EW theory with massive & massless gauge bosons
- factorization of IR singularities shown explicitly ✓

Outlook

- extend to non-Abelian models: SU(2), SU(N)
- consider effects from symmetry breaking: Higgs contributions, $M_Z > M_W$
- 4-fermion scattering amplitude
- full electroweak standard model